Keynesian Micromanagement*

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Abstract

We develop a general theory of *optimal* sector-specific government spending in a disaggregated production economy, a policy practice we label *Keynesian Micromanagement*. To study optimal sector-specific spending, we build a novel multi-sector model which combines search-and-matching frictions at the level of individual goods with realistic heterogeneity in price rigidity, input-output linkages and elasticities of substitution in production. Crucially, we establish that such granular search frictions are isomorphic to endogenous movements in sectoral total factor productivity. Our novel optimal policy principle directly exploits this equivalence between sectoral search frictions and endogeneous productivity changes. In particular, we show that sector-specific government spending should deviate from the frictionless benchmark given by the Samuelson rule to the extent and in the direction that sectoral spending affects aggregate measured total factor productivity.

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1 Introduction

From the Covid-19 pandemic to the Russian invasion of Ukraine, the world economy has been shaken by shocks that are both very large in magnitude and highly asymmetric in their effect on different sectors and industries. Designing an appropriate response to such shocks therefore requires broadening our toolkit beyond models which feature aggregate policy tools, such as the central bank interest rate or total government spending. In this paper we develop a general theory of *optimal* sector-specific government spending, a policy practice we label *Keynesian Micromanagement*.

To study optimal sector-specific spending, we build a novel multi-sector model which combines search-and-matching frictions at the level of individual goods with realistic heterogeneity in price rigidity, input-output linkages and elasticities of substitution in production. Our model generates involuntary spare capacity in all sectors of the economy, which is generally inefficient and should be corrected with sector-specific spending. Extra spending directed to a specific sector has two opposing effects: on the one hand, it increases capacity utilisation in that sector; on the other hand, it raises the cost of search for households. Crucially, we establish that the interaction between the two effects is isomorphic to endogenous movements in sectoral total factor productivity. Whenever the capacity effect dominates the cost of search effect the sectoral productivity rises, and *vice versa*.

Our novel optimal policy principle directly exploits this equivalence between sectoral search frictions and endogeneous productivity changes. In particular, we show that sector-specific government spending should deviate from the frictionless benchmark given by the Samuelson rule to the extent and in the direction that sectoral spending affects aggregate measured total factor productivity. The latter can be calculated by using Hulten's theorem to aggregate sector-specific endogenous productivities. Moreover, we show that the effect of sector-specific spending on aggregate endogenous productivity and hence optimal stimulus can be approximated using observable variables, such as sectoral capacity utilisation, Domar weight, cost pass-though to prices and production function primitives.

We also compare and contrast optimal sector-specific stimulus near efficiency versus around a point with distortions. Locally around efficiency, optimal policy depends only on sectoral capacity utilization, Domar weights and cost pass-though to prices. The exact details on input-output linkages and input-specific elasticities of substitution become relevant only when optimal policy is to be computed around a point with pre-existing distortions.

2 Model

2.1 Overview

The model is static. The economy has three types of agents: households, firms and the government. The representative household gains utility from consuming sector-specific goods, as well as the numeraire good, and supplies $\overline{L}$ units of labor inelastically.

Firms are subdivided into $(N+1)$ sectors, indexed by $i \in \{0, 1, ..., N\}$. Sector $i = 0$ is the *labor union*, which purchases the $\overline{L}$ units of labor from the households and sells them to the other sectors as an intermediate input. Firms in the remaining sectors $i \geq 1$ use labor purchased from the labor union, as well as intermediate inputs purchased from the other sectors, to produce a sector-specific good, which can either be sold to the households as a final good, or as an intermediate good to firms in other sectors of the economy.

The government is a fiscal authority, which can implement fiscal policy through sectoral government consumption of goods produced in all sectors, apart from the labor union. The government finances its sectoral consumption through a lump-sum tax levied on households.

Crucially, trade in all markets is subject to search-and-matching frictions. Households, the government and firms need to make costly visits to firms in order to make purchases of final and intermediate goods, respectively. Importantly, not all visits are successful. At the same time, the search friction also means that firms cannot sell all of their productive capacity, which means there is involuntary *spare capacity*. Importantly, the spare capacity in the labor union sector is a measure of involuntary *unemployment*.

2.2 Search-and-matching frictions in sectoral goods markets

The market for each sectoral good is subject to search-and-matching frictions. In particular, given the endogenous productive capacity of sector i (K_i) and the total number of visits made to that sector (V_i), the resulting number of sales is given by the following matching function:

$$h^i(K_i, V_i), \quad i = 0, 1, ..., N \tag{1}$$

where the total number of visits V_i comprises of the visits by households (V_i^H), visits by the government (V_i^G), as well as visits by representatives of firms that use i's output as in intermediate input (V_i^F):

$$V_i = V_i^H + V_i^G + V_i^F, \quad i = 0, 1, ..., N \tag{2}$$

We assume that the matching function is strictly increasing and constant returns to scale in both of its inputs. Further, we impose the restriction that $h^i(K_i, V_i) < \min\{K_i, V_i\}$, so that there is always some unutilised capacity and some unsuccessful visits.

A crucial variable that characterizes the state of each goods market is the sectoral goods market *tightness* (x_i), which is defined by the ratio visits to capacity:

$$x_i \equiv \frac{V_i}{K_i}, \quad i = 0, 1, ..., N. \tag{3}$$

Intuitively, x_i is a measure of how congested a given goods market is. When tightness is large, there are a lot of visits relative to capacity, so the goods market is congested, and *vice versa*.

Abstracting from uncertainty, each unit of productive capacity is sold with probability $f_i(x_i) \equiv \frac{h^i(K_i,V_i)}{K_i} = h^i(1, x_i)$, where $f' > 0$. Intuitively, the probability of selling a unit of productive capacity is higher in a tighter goods market, and *vice versa*. Similarly, each visit is successful with probability $q_i(x_i) \equiv \frac{h^i(K_i,V_i)}{V_i} = h^i(1/x_i, 1)$, where $q' < 0$. The interpretation is once again intuitive: the probability of a successful visit is lower in a tighter goods market, and *vice versa*.

Each visit to a firm in sector i costs $\rho_i \in (0,1)$ units of i's output. Since every such visit is successful with probability $q_i(x_i)$, total sales in sector i are given by $q_i(x_i)V_i$ [1]. Therefore, the total number of sector i's goods that need to be purchased (inclusive of the cost of visits) in order to obtain one unit is given by $[1+\gamma_i(x_i)]$, where

$$\gamma_i(x_i) \equiv \frac{\rho_i}{f_i(x_i) - \rho_i x_i}, \quad i = 0, 1, ..., N. \tag{4}$$

represents a *congestion wedge* introduced by search-and-matching frictions that strictly rises in goods market tightness, such that $\gamma_i'(x_i) > 0, \forall i, \forall x_i \in (0, x_i^m)$.[2] Intuitively, a tighter goods market lowers the probability of a successful visit, increasing the expected number of visits required for a successful purchase, thus raising the total cost of visits and thus implying a larger congestion wedge.

2.3 Households

Households derive utility from consuming goods produced in all sectors $i \geq 1$, apart from the labor union $(i = 0)$. Each sectoral good $i \geq 1$ can either be privately purchased (C_i) or provided by the government (G_i). Total consumption of sectoral good $i \geq 1$ by households is given by the following final demand aggregator:

$$D^i(C_i, G_i), \quad i = 1, ..., N. \tag{5}$$

We assume that each final demand aggregator is strictly increasing, differentiable and constant returns to scale in both arguments.

In addition to the sectoral goods, households derive utility from consumption of the numeraire good M, which is traded in a frictionless market and is in fixed exogenous supply $\overline{M}$, received as an endowment. All in all, the representative household has the following utility function:

$$U\left[D^1(C_1, G_1), ..., D^N(C_N, G_N)\right] + \mathcal{V}(M) \tag{6}$$

where $U[D^1, ..., D^N]$ is continuous, differentiable, increasing, strictly quasi-concave and exhibits constant returns to scale in all arguments; $\mathcal{V}(.)$ is increasing, differentiable and concave. The representative household chooses $\left\{\{C_i\}_{i=1}^N, M\right\}$ to maximize the utility in (6), subject to the following budget constraint:

$$\sum_{i=1}^{N} P_i[1+\gamma_i(x_i)]C_i + M \leq W\overline{L} + \overline{M} - T \tag{7}$$

where W is the wage, $\overline{L}$ is the fixed inelastic labor supply, T is the lump-sum tax levied by the government; P_i is the *posted* price of good i, which is the price households see in stores. However, the total cost of purchasing C_i units of good i also includes the cost of making visits, summarised by the congestion wedge $[1+\gamma_i(x_i)]$. We therefore refer to $P_i[1+\gamma_i(x_i)]$ as the *effective* price of good i.

[1] This includes sales to households, government and other firms

[2] We restrict the admissible values of tightness for each sector to $(0, x_i^m)$, where x_i^m is given by the condition $f_i(x_i^m) = \rho_i x_i^m$; this restriction ensures that the supply of each sector good, net of the cost of visits, remains positive.

Combining the first order conditions for C_i and M, we get the following household optimality condition:

$$\frac{\partial U}{\partial D^i}\frac{\partial D^i}{\partial C_i} = P_i[1+\gamma_i(x_i)]\mathcal{V}'(M), \quad i = 1, ..., N. \tag{8}$$

2.4 Firms

Firms in any sector $i \geq 0$ have access to the following production function, which transforms labor and intermediate inputs into productive capacity K_i:

$$K_i = F^i\left[L_i, \{Z_{ij}\}_{j=0}^N\right], \quad i = 0, 1, ..., N \tag{9}$$

where L_i is the labor input and Z_{ij} is intermediate inputs purchased by sector i from sector j. We assume that each production function is strictly quasi-concave, increasing, differentiable, continuous and exhibits constant returns to scale in all inputs used.

Due to our assumption regarding the existence of a labor union, the labor union sector ($i = 0$) is the only sector that directly purchases labor and does not purchase intermediate inputs ($Z_{0j} = 0, \forall j \geq 0$). Direct labor input is zero for all other sectors ($L_i = 0, \forall i \geq 1$), whereas any empirical use of labor inputs is modelled as an intermediate input purchased from the labor union sector ($Z_{i0}, \forall i \geq 1$).

Firms in sector i choose labor and intermediate inputs in order to minimize the total cost of production:

$$WL_i + \sum_{j=0}^{N} P_j[1+\gamma_j(x_j)]Z_{ij} \tag{10}$$

subject to the production function in (9). Note that intermediate inputs need to be purchased in markets with search-and-matching frictions, which is why the relevant price of purchasing intermediate inputs from a sector $j \geq 0$ is the *effective* price of sector j, which includes the congestion wedge $[1+\gamma_j(x_j)]$. This cost minimization problem delivers the marginal cost function for each sector:

$$MC_i\left[W, \{P_j[1+\gamma_j(x_j)]\}_{j=0}^N\right] \tag{11}$$

Due to the properties of the production function, the marginal cost function is common for all firms in a given sector, increasing, continuous and exhibits constant returns to scale in the wage and sectoral *effective* prices.

We assume that firms in all sectors have no market power and there is free entry, which is summarised by the following zero profit condition for each sector:

$$P_i f_i(x_i)K_i = MC_i K_i, \quad i = 0, 1, ..., N \tag{12}$$

Notice that the zero profit condition takes into account that only a fraction $f_i(x_i)$ of productive capacity is sold.

Importantly, in all sectors not all productive capacity is utilized, which means there is involuntary spare capacity. We formally define and use the following measure of (involuntary) spare capacity:

Definition 1 (Spare capacity). *Define the rate of spare capacity in sector $i \geq 0$, denoted S_i, as one minus the fraction of productive capacity sold:*

$$S_i \equiv 1 - f_i(x_i), \quad i = 0, 1, ..., N \tag{13}$$

Crucially, spare capacity in the labor union sector, given by S_0, is a measure of involuntary unemployment.

2.5 Government

The government is able to choose its level of consumption in all sectors apart from the labor union. Given the set of government consumptions $\{G_i\}_{i=1}^N$, it collects the following lump-sum tax from the households:

$$T = \sum_{i=1}^{N} P_i[1 + \gamma_i(x_i)]G_i. \tag{14}$$

Notice that we the government needs to purchase its desired consumption levels in sectoral goods markets with search-and-matching frictions. This is why the relevant prices for calculating the total lump-sum tax are the *effective* prices for each sector, which include the sectoral congestion wedges.

2.6 Closing the model and equilibrium

In addition to the agent optimality and policy conditions above, equilibrium in our economy is pinned down by market clearing conditions. First, there is market clearing in the numeraire good market:

$$M = \overline{M} \tag{15}$$

Second, there is market clearing in the labor market:

$$L_0 = \overline{L} \tag{16}$$

Notice that since the labor union sector ($i = 0$) is the only one that purchases labor directly from households, market clearing in the labor market requires that the labor demand from the union sector (L_0) equals the exogenous inelastic labor supply ($\overline{L}$).

Third, there is goods market clearing condition for every sector $i \geq 0$, which is more nuanced and requires a more extensive explanation, which we provide here. The total demand for any sector $i \geq 0$ comprises of households' consumption (C_i), government consumption (G_i), as well the use of that sector's output as an intermediate input by other sectors $\left(\sum_{j=1}^N Z_{ji}\right)$. The sectoral supply that is available to satisfy that total demand is the productive capacity K_i, which needs to further adjusted by two wedges. First, only a fraction of $f_i(x_i)$ of productive capacity is sold; second, of $f_i(x_i)K_i$ that is sold, a fraction is further wasted on the cost of making visits to that sector. Thus, only $\frac{f_i(x_i)}{1+\gamma_i(x_i)}K_i$ is left to satisfy the total demand for that sector. Defining $A_i(x_i) \equiv \frac{f_i(x_i)}{1+\gamma_i(x_i)}$, we can therefore write sectoral goods market clearing condition as:

$$C_i + G_i + \sum_{j=1}^{N} Z_{ji} = A_i(x_i)K_i, \quad i = 0, 1, ..., N \tag{17}$$

Note that for the labor union sector ($i = 0$) both households' and government consumption is zero ($C_0 = G_0 = 0$), whereas $\{Z_{j0}\}_{j=1}^{N}$ represent the use of labor by other sectors.

Moreover, we can also rewrite the zero profit condition in (13) in terms of $A_i(x_i)$:

$$P_i[1 + \gamma_i(x_i)] = \frac{1}{A_i(x_i)} MC_i. \tag{18}$$

Observing the conditions in (17) and (18), as well as the remaining optimality and market clearing conditions, it can be seen that our model is equivalent to a frictionless model with competitive (effective) prices, augmented with *endogenous* sectoral productivities $\{A_i(x_i)\}_{i=0}^{N}$.

Moreover, each endogenous sectoral productivity $A_i(x_i)$ has an inverted U-shaped relationship with that sector's goods market tightness:

Lemma 1. *For every sector $i \geq 0$ there exists tightness $x_i^* \in (0, x_i^m)$, such that:*

(i) $A_i'(x_i^*) = 0$;

(ii) $A_i'(x_i) > 0, \quad \forall x \in (0, x_i^*)$

(iii) $A_i'(x_i) < 0, \quad \forall x \in (x_i^*, x_i^m)$

Moreover, for every sector $i \geq 0$, x_i^* is the constrained-efficient level of tightness.

Intuitively, an increase in tightness has two opposing effects on the endogenous sectoral productivity. On the one hand, it increases the fraction of capacity that is sold, $f_i(x_i)$; on the other hand, it lowers the probability of a successful visit, thus increasing the total cost of visits and the congestion wedge $[1 + \gamma_i(x_i)]$. For sufficiently low levels of tightness, the first effect dominates and $A_i(x_i)$ rises in tightness; for sufficiently high levels of tightness, the second effect dominates and $A_i(x_i)$ falls in tightness. At the constrained-efficient level of tightness, the two effects exactly offset each other and the endogenous sectoral productivity in maximized.[3]

Fully solving the model therefore requires keeping track of the levels sectoral goods market tightness. However, without further assumptions, sectoral prices and levels of tightness are not separately identified, which is a common feature of models with random search, like ours.[4] We resolve this indeterminacy by imposing a general rule for *posted* prices $\{P_i\}_{i=0}^{N}$:

[3]We formally solve for the problem of the social planner, constrained by the matching function and search costs, in Appendix

[4]In models of search-and-matching in the labor market, this indeterminacy can be resolved in a number of ways. For example, one could additionally assume Nash Bargaining over the wage between firms and workers. Alternatively, one could assume fixed wages or wages that are proportional to productivity.

Assumption 1 (Posted prices). *Posted price for firms in a sector $i \geq 0$ are given by:*

$$P_i = \mathcal{P}_i(MC_i), \quad i = 0, 1, ..., N \tag{19}$$

where $\mathcal{P}_i' \geq 0$, $\mathcal{P}_i'' \leq 0$, with the additional restriction that $\frac{MC_i}{\mathcal{P}_i(MC_i)} \in (0, 1)$, $i \geq 0$.

We can now formally define equilibrium in our economy:

Definition 2 (Equilibrium). *Equilibrium is a collection of allocations $\left\{\left\{C_i, L_i, \{Z_{ij}\}_{j=1}^N\right\}_{i=0}^N, M\right\}$, sectoral posted prices $\{P_i\}_{i=0}^N$, sectoral levels of tightness $\{x_i\}_{i=0}^N$ and wage W, such that, given the sectoral government consumptions $\{G_i\}_{i=1}^N$ and the posted pricing rules $\{\mathcal{P}_i(.)\}_{i=0}^N$, agent optimality and market clearing conditions are satisfied.*

Crucially, the mappings $\{\mathcal{P}_i(.)\}_{i=0}^N$ nests a variety of different approaches to separate out equilibrium prices and levels of tightness. However, apart from very specific special cases, our pricing rules do not imply equilibrium levels of tightness that are constrained efficient.

For example, consider the following mapping: $\mathcal{P}_i(MC_i) = (1/f_i(x_i^*))MC_i, \forall i \geq 0$. Combining this mapping with the zero-profit condition in (13) implies that $f_i(x) = f_i(x^*) \Leftrightarrow x_i = x_i^*, \forall i \geq 0$. Therefore, this specific pricing rule does deliver sectoral levels of rightness that are always constrained efficient. On the other hand, consider an alternative pricing rule: $\mathcal{P}_i(MC_i) = (1/f_i(x_i^*))MC_i^{1-r}$, where $r \in (0, 1)$ is a constant. Then the zero-profit condition implies that $f_i(x) = f_i(x^*)MC_i^r, \forall i \geq 0$. In this case, sectoral levels of tightness are constrained efficient only in *specific* states of the world, where all sectoral marginal costs are exactly equal to one.

Therefore, under our general pricing rule in (19), sectoral levels of tightness need not be constrained efficient in equilibrium. There is therefore a potential room for policy that can improve the equilibrium outcome from a welfare perspective. In the next section we study how sectoral government consumptions $\{G_i\}_{i=1}^N$ can be optimally chosen to maximise equilibrium welfare.

3 Optimal fiscal policy

3.1 Optimal fiscal policy in the baseline model

Equilibrium outcomes introduced in Definition 2 are conditional on a specific set of government consumptions $\mathbb{G} \equiv \{G_i\}_{i=1}^N$. Optimal fiscal policy amounts to *choosing* $\mathbb{G}$ that maximises equilibrium welfare. Formally, the optimal fiscal policy can be written as:

$$\max_{\{G_i\}_{i=1}^N} U\left[D^1(C_1(\mathbb{G}), G_1), ..., D^N(C_N(\mathbb{G}), G_N)\right] \tag{20}$$

where $C_i(\mathbb{G})$ is the equilibrium household consumption of sector i's output, conditional on sectoral government consumptions $\mathbb{G}$.[5]

[5] We drop $\mathcal{V}(M)$ from the objective function, as its equilibrium value is $\mathcal{V}(\overline{M})$, which is independent of the choice of government consumption.

One can see that government consumption in any sector $k \geq 1$ affects equilibrium welfare through two channels. First, it directly affects the final demand for sector k's output: $\frac{\partial U}{\partial D^k}\frac{\partial D^k}{\partial G_k}$. Second it can, in general, indirectly affect households consumption in any other sector $i \geq 1$: $\frac{\partial U}{\partial D^i}\frac{\partial D^i}{\partial C_i(\mathbb{G})}\frac{\partial C_i(\mathbb{G})}{\partial G_k}$. In order to understand the last effect, $\frac{\partial C_i(\mathbb{G})}{\partial G_k}$, let us combine it with sector i's goods market clearing condition:

$$\frac{\partial C_i(\mathbb{G})}{\partial G_k} = \overbrace{A_i'(x_i)\frac{\partial x_i(\mathbb{G})}{\partial G_k}K_i + A_i(x_i)\frac{\partial K_i(\mathbb{G})}{\partial G_k}}^{\text{Supply-side effect}} \overbrace{- \frac{\partial G_i}{\partial G_k} - \sum_{j=1}^{K}\frac{\partial Z_{ji}(\mathbb{G})}{\partial G_k}}^{\text{Demand-side effect}} \tag{21}$$

Additional sectoral government consumption has both supply- and demand-side effects. On the supply side, there are two channels. First, in our framework with a common labor market and input-output linkages, spending on sector k can affect tightness in any other sector i. This change in tightness of sector i then changes the endogenous sectoral productivity of that sector, though the direction is ambiguous: if x_i is below the constrained efficient value, endogenous sector productivity rises in tightness, and *vice versa*. Second, spending on sector k can also affect the productive capacity of any other sector i. This is because of reallocation of factors of production across sectors due to movements in relative *effective* prices. All in all, the direction of the supply-side effect of additional government consumption is ambiguous.

On the demand side, there are also two channels. First, iff $k = i$, there is a mechanical effect of crowding out household sectoral consumption with government sectoral consumption in the same sector. Second, additional government consumption on sector k can affect the intermediate input demand for output of any sector i, due to input-output linkages. The direction of the second effect is generally ambiguous and depends on reallocation on factors of production across sectors and initial levels of tightness across sectors.

All in all, although the direct own effect of additional government consumption in sector k on final demand in sector k is straightforward, the additional indirect effect on household consumption in any other sector i is multi-faceted and generally ambiguous in its direction.

Nonetheless, it is possible to obtain a tractable formulation for the first order condition that optimal government consumption in any sector $i \geq 1$ must satisfy:

Proposition 1. *First order condition for optimal government consumption of sector i's output (G_i) is given by:*

$$\text{FOC}(G_i): \qquad \omega_i^G[1 - MRS_i^{GC}] = \sum_{t=0}^{N}\lambda_t\frac{d\log A_t(x_t)}{d\log x_t}\frac{\partial \log x_t}{\partial \log G_i}, \qquad i = 1, ..., N \tag{22}$$

where

- $MRS_i^{GC} \equiv \frac{\partial D_i/\partial G_i}{\partial D_i/\partial C_i}$ *is the marginal rate of substitution between government and households' consumption of sector i's output*
- $\omega_i^G \equiv \frac{P_i[1+\gamma_i(x_i)]G_i}{\sum_{j=1}^{N}P_j[1+\gamma_j(x_j)](C_j+G_j)}$ *is nominal government expenditure on sector i as a share of nominal GDP*
- $\lambda_i \equiv \frac{P_i[1+\gamma_i(x_i)]A_i(x_i)K_i}{\sum_{j=1}^{N}P_j[1+\gamma_j(x_j)](C_j+G_j)}$ *is the Domar weight (sales share) of sector i*

We can analyze the first order condition through the lens of direct and indirect effects we have outlined above. First, the left-hand side features a difference between the marginal rate of substitution (MRS) across government and household consumption in the same sector and one. This represents a balance between the direct effect of G_i on total final demand for sector i, and the indirect demand-side effect of mechanical crowding out of household consumption with government consumption in the same sector. Second, the right hand side is a manifestation of the indirect supply-side effect of government consumption in sector i on levels of tightness, and hence the endogenous productivity, in all other sectors. Crucially, none of the effects related to the reallocation of factors of production across sectors feature in the first order condition. This is a consequence of the zero-profit condition and hence competitive equilibrium *effective* prices. The latter implies that, in the aggregate, allocation of factors is (constrained) efficient across sectors and the policy intervention with government consumption should not change it.

In fact, we can get a better understanding of the *aggregate* supply-side effect of government consumption by linking changes in aggregate measured TFP to the sectoral endogenous productivities induced by the fiscal interventions. This can be done using the canonical aggregation theorem of Hulten (1978):

Lemma 2 (Hulten, 1978)**.** *Let $TFP = GDP/L$ be the measured aggregate TFP of our economy, then:*

$$d \log TFP = \sum_{t=0}^{N} \lambda_t d \log A_t(x_t) \tag{23}$$

where λ_t is the Domar weight (sales share) of sector t

All in all, up to first order, the change in aggregate measured TFP is a just a a sum of changes in endogenous sectoral productivities, weighted by sectoral Domar weights. We can now combine Lemma 2 with Proposition 1 to obtain an optimality condition for sectoral government consumption in terms of aggregate measured TFP:

Theorem 1. *Optimal government consumption of sector i's output (G_i) satisfies:*

$$\underbrace{MRS_i^{GC}}_{\text{Samuelson rule}} = 1 - \frac{1}{\omega_i^G} \times \frac{d \log TFP}{d \log G_i}, \qquad i = 1, ..., N \tag{24}$$

where

- *MRS_i^{GC} is the marginal rate of substitution between government and households' consumption of sector i's output*
- *ω_i^G is nominal government expenditure on sector i as a share of nominal GDP*
- *TFP is aggregate measured total factor productivity.*

Theorem 1 is the main theoretical result of our paper, which provides a tractable generalization to the canonical Samuelson rule for a disaggregated multi-sector economy with frictions. In particular, for every sectoral government consumption, it is optimal to deviate from the (sector-specific) Samuelson rule to the extent to which it changes aggregate measured TFP. Since MRS falls in government consumption, the fiscal authority should exceed the Samuelson-prescribed level of sectoral government consumption if it increases TFP.

In an economy with no search frictions there are no endogenous sectoral productivities, so that $d\log TFP/d\log G_i = 0, \forall i \geq 1$, and the theorem collapses to the canonical Samuelson rule for every sector. Moreover, even with search frictions, the theorem collapses to the canonical Samuelson rule at the point when all sectoral tightness levels are at the constrained efficient level, since $d\log TFP = \sum_{t=0}^{N} \lambda_t d\log A_t(x_t^*) = \sum_{t=0}^{N} \lambda_t \underbrace{A_t'(x_t^*)}_{=0} \frac{dx_t}{A_t(x_t^*)} = 0$.

Another notable special case arises whenever household and government consumption are perfect substitutes in every sector, so that $MRS_i^{GC} = 1, \forall i \geq 1$. In this case optimal fiscal policy collapses to $d\log TFP/d\log G_i = 0, \forall i \geq 1$, so that sectoral government consumptions should be chosen to maximize aggregate measured TFP. An outcome which satisfies this optimality condition is $x_i = x_i^*, \forall i \geq 0$, so that the government is to target constrained efficient levels of tightness in every sector.

4 Optimal fiscal policy: an approximation

4.1 Functional forms

We now specify the functional forms for a number of key mappings of our model. This allows us to characterise a first-order approximation to optimal fiscal policy, parameterized by deep structural primitives.

For the final demand aggregators D^i we assume those to take a constant elasticity of substitution form, with sector-specific elasticities of substitution across households and government consumption:

Assumption 2 (Demand aggregator)**.** *The final demand aggregator for sector i is given by:*

$$D^i(C_i, G_i) = \left[(1-\delta_i)^{\frac{1}{\epsilon_i}} C_i^{\frac{\epsilon_i - 1}{\epsilon_i}} + \delta_i^{\frac{1}{\epsilon_i}} G_i^{\frac{\epsilon_i - 1}{\epsilon_i}}\right]^{\frac{\epsilon_i}{\epsilon_i - 1}}, \quad i = 1, ..., N \tag{25}$$

where $\epsilon_i > 0$ is the elasticity of substitution between private and government provision of sector i's output, $\delta_i \in (0,1)$ is the relative bias for government provision of sector i's output

The elasticity of substitution parameter $\epsilon_i > 0$ governs the degree to which government provision of a sectoral good can act as a substitute for the privately purchased version of the same good. Naturally, higher values of ϵ_i indicate that privately and publicly provided versions of the same good are closer substitutes, with $\epsilon_i \to \infty$ corresponding to the case of perfect substitutes.

Under such functional form assumption about final demand aggregator, the marginal rate of substitution between government and household consumption of sectoral good $i \geq 1$ takes the following form:

$$MRS_i^{GC} = \left(\frac{\delta_i}{1-\delta_i}\right)^{\frac{1}{\epsilon_i}} \left((\frac{G_i}{C_i})\right)^{-\frac{1}{\epsilon_i}}, \quad i = 1, ..., N \tag{26}$$

We also make the following function form assumption for the utility over final demands U:

Assumption 3 (Utility function). *The utility function over demands over sectoral goods is given by:*

$$U = \sum_{i=1}^{N} \frac{[D^i(C_i, G_i)]^{1-\sigma} - 1}{1-\sigma} \tag{27}$$

where $\sigma > 0$.

This specific form of utility implies that the household optimality condition for sectoral consumption from (8) can be written as:

$$G_i^{-\sigma}\left[D^i\left(\left(\frac{G_i}{C_i}\right)^{-1}, 1\right)\right]^{-\sigma}\frac{\partial D^i}{\partial C_i}\left(\frac{G_i}{C_i}\right) = P_i[1+\gamma_i(x_i)]\mathcal{V}'(\overline{M}), \quad i = 1, ..., N \tag{28}$$

where we have used the fact that D^i is homogenous of degree one in its inputs. The formulation above allows to express G_i as a function of $(G_i/C_i), x_i$ and P_i. This will be extremely convenient in the derivation of approximate fiscal policy, as the marginal rate of substitution is a function of (G_i/C_i).

As for the sector-specific matching function, we assume that it takes the following Cobb-Douglas form:

Assumption 4 (Matching function). *The matching function for sector i admits the following form:*

$$h^i(K_i, V_i) = \psi_i^h \times K_i^{\eta_i} V_i^{1-\eta_i}, \quad i = 0, 1, ..., N \tag{29}$$

where $\eta_i \in (0,1)$ *is the sector-specific elasticity between the number of matches and productive capacity,* $\psi_i^h > 0$ *is a sector-specific match-efficiency parameter*

Under this form of the matching function, the probability of successful sale satisfies $f_i(x_i) = \psi_i^h x_i^{1-\eta_i}$ and $f_i'(x_i) = (1-\eta_i)\psi_i^h x_i^{-\eta_i} = (1-\eta_i) f_i(x_i)/x_i$, and similarly for the probability of successful visit, where $q_i(x_i) = \psi_i^h x_i^{-\eta_i}$ and $q_i'(x_i) = -\eta_i \psi_i^h x_i^{-\eta_i - 1} = -\eta q_i(x_i)/x_i$. Such form of the matching function also delivers a tractable condition for the constrained efficient level of tightness x_i^*:

$$\eta_i \gamma_i(x_i^*) = 1 - \eta_i, \quad i = 0, 1, ..., N \tag{30}$$

Recall that in Assumption 1 we specified a general class of rules for posted prices, in the form of a mapping from the marginal cost to the posted price. Here we consider a more specific class of rules for posted prices, which features a constant elasticity of pass-through from the marginal cost to prices:

Assumption 5 (Posted prices). *Posted price for firms in a sector* $i \geq 0$ *are given by:*

$$P_i = \psi_i^p \times MC_i^{1-r_i}, \quad i = 0, 1, ..., N \tag{31}$$

where $r_i \in (0,1)$ *is the pass-through of marginal cost to the posted price,* $\psi_i^p > 0$ *is a parameter*

Combining this assumption regarding posted prices with the zero-profit condition in (13), we get that:

$$f_i(x_i) = (\psi_i^p)^{-1} \times MC_i^{r_i} \qquad\qquad P_i = (\psi_i^p)^{1/r_i} \times (f_i(x_i))^{\frac{1-r_i}{r_i}}, \quad i = 0, 1, ..., N \tag{32}$$

Two points should be noted. First, as $r_i \to 0$, the probability of a a successful sale converges to a (non state-contingent) constant $(\psi_i^p)^{-1}$, which in turn means that sectoral tightness converges to a constant as well. This is because under full pass-through from marginal cost to posted prices ($r_i = 0$) none of the market clearing function is performed by tightness, so it remains a constant. Second, one can see that for $r_i \in (0, 1)$ the sectoral posted price can be expressed as a function of sectoral tightness only.

Our final function form assumption concerns the form of the production, which we set to be of the constant elasticity of substitution form:

Assumption 6 (Production function)**.** *Productive capacity is manufactured using the following production function:*

$$K_i = \left[\overline{\alpha}_i^{\frac{1}{\theta_i}} L_i^{\frac{\theta_i - 1}{\theta_i}} + \sum_{j=0}^{N} \overline{\omega}_{ij}^{\frac{1}{\theta_i}} Z_{ij}^{\frac{\theta_i - 1}{\theta_i}} \right]^{\frac{\theta_i}{\theta_i - 1}}, \quad i = 0, 1, ..., N \tag{33}$$

where $\theta_i > 0$ is the elasticity of substitution across inputs, $\overline{\alpha}_i, \{\overline{\omega}_{ij}\}_{i=0,j=0}^N \in [0, 1]$ are input-specific bias parameters, where $\overline{\alpha}_i + \sum_{j=0}^N \overline{\omega}_{ij} = 1, \forall i \geq 0$.

Recall that the labor union sector ($i = 0$) is the only one that purchases labor directly and does not purchase intermediate inputs, so $\overline{\alpha}_0 = 1, \overline{\omega}_{0j} = 0, \forall j \geq 0$. As for all the other sectors $i \geq 1$, we have $\overline{\alpha}_i = 0, \sum_{j=0} \overline{\omega}_{ij} = 1, \forall i \geq 1$. Such form of the production function also produces tractable expressions for the input-output cost shares in non labor union sectors $i \geq 1$:

$$\omega_{ij} \equiv \frac{P_j[1 + \gamma_j(x_j)]Z_{ij}}{MC_i K_i} = \overline{\omega}_{ij} \left[\frac{P_j[1 + \gamma_j(x_j)]}{MC_i} \right]^{1-\theta_i}, \quad \forall i \geq 1, \forall j \geq 0 \tag{34}$$

Naturally, when the production function is Cobb-Douglas ($\theta_i = 1$) the equilibrium input-output cost shares are fixed parameters.

4.2 Optimal fiscal policy under the functional forms

Our next step is to apply the functional form assumptions above to the first order condition for optimal sectoral government consumption in Proposition 1.

On the left hand side of Proposition 1, the only term that explicitly depends on the functional forms is the marginal rate of substitution (MRS), whose form under the functional forms is already given in (26). We therefore only need to go through the terms of the right hand side.

First, the Domar weights (sales shares): $\{\lambda_i\}_{i=0}^N$. Taking the goods market clearing condition (17) for a sector $i \geq 0$, multiplying both sides by $P_i[1 + \gamma_i(x_i)]$ and dividing both sides by nominal GDP

$\sum_{j=1}^{N} P_j[1+\gamma_j(x_j)](C_j+G_j)$ yields:

$$\lambda_i = \omega_i^{CG} + \sum_{j=1}^{N} \omega_{ji}\lambda_j, \quad i = 0, 1, ..., N \tag{35}$$

where

$$\omega_o^{CG} = 0, \qquad \omega_i^{CG} \equiv \frac{P_i[1+\gamma_i(x_i)](C_i+G_i)}{\sum_{j=1}^{N} P_j[1+\gamma_j(x_j)](C_j+G_j)}, \forall i \geq 1 \tag{36}$$

are the final consumption shares and ω_{ij} are the input-output cost shares, whose form under the functional form assumptions is given in (34).

Second, the elasticity of endogenous sectoral productivity with respect to sectoral tightness: $\frac{d \log A_i(x_i)}{d \log x_i}$. Applying the functional form assumption regarding matching function it can be shown that:

$$\frac{d \log A_i(x_i)}{d \log x_i} = (1-\eta_i) - \eta_i \gamma_i(x_i), \quad i = 0, 1, ..., N. \tag{37}$$

Third, the cross elasticity of tightness in sector t to government consumption in sector i: $\frac{\partial \log x_t}{\partial \log G_i}$. This derivation is more involved and we will do it in two steps.First consider the zero profit condition for a sector t in (13), take logs from both sides and take a derivate with respect to G_i from both sides. It can be shown that:

$$\frac{1-\eta_t}{r_t}\frac{\partial \log x_t}{\partial \log G_i} = \alpha_t \frac{\partial \log W}{\partial \log G_i} + \sum_{j=0}^{N} \underbrace{\omega_{tj}\left(1 - r_j + \frac{\eta_j \gamma_j(x_j)}{1-\eta_j} r_j\right)}_{\equiv \hat{\omega}_{tj}} \frac{1-\eta_j}{r_j}\frac{\partial \log x_j}{\partial \log G_i}, \quad t = 0, 1, ..., N \tag{38}$$

Stacking the equations above and rearranging delivers:

$$\frac{\partial \log x_t}{\partial \log G_i} = \frac{r_t}{1-\eta_t}\left[(I-\hat{\Omega})^{-1}\right]_{t0} \frac{\partial \log W}{\partial \log G_i} \tag{39}$$

where $[\hat{\Omega}]_{tj} = \hat{\omega}_{tj} \equiv \omega_{tj}\left(1 - r_j + \frac{\eta_j \gamma_j(x_j)}{1-\eta_j} r_j\right)$ are the *congestion-adjusted* input-output cost shares.

Three things should be noted. First, *ceteris paribus*, tightness in sectors with lower marginal cost pass-through r_t responds more to a spending interventions in sector i. This is because low pass-through prevents market clearing through prices, forcing sector t to clear demand and supply through tightness. Second, *ceteris paribus*,tightness in sectors with lower matching function elasticity η_t responds more to a spending interventions in sector i. This is because low η_t implies that larger changes in x_t are needed to obtain the same change in $f_t(x_t)$, which is what needs to adjust for the zero-profit condition to hold. Third, *ceteris paribus*,tightness in sectors with higher *congestion-adjusted Leontief distance* to the labor union sector $\left[(I-\hat{\Omega})^{-1}\right]_{t0}$ responds more to a spending interventions in sector i. Intuitively, $\left[(I-\hat{\Omega})^{-1}\right]_{t0}$ measures a sector's *total* network-adjusted exposure to the labor union, which includes own exposure, exposure of its suppliers, exposure of suppliers' suppliers' and so on. Whenever a sector's total exposure to the labor union is larger, its tightness would also be more sensitive to movements in the wage.

To complete the derivation of $\frac{\partial \log x_t}{\partial \log G_i}$, we need to provide an expression for $\frac{\partial \log W}{\partial \log G_i}$. Consider the households budget constraint in equilibrium: $\sum_{j=1}^{N} P_j[1+\gamma_j(x_j)](C_j+G_j) = W\overline{L}$. Taking logs from both sides, finding a derivative with respect to G_i and using (39):

$$\frac{\partial \log W}{\partial \log G_i} = \frac{\omega_i^{CG}\left(1-\frac{\overline{\delta}_i}{1+(G_i/C_i)}\right)}{1-\sum_{j=1}^{N}\omega_j^{CG}\left(1-\frac{\overline{\delta}_i/\sigma}{1+(G_i/C_i)}\right)\left(1-r_j+\frac{\eta_j\gamma_j(x_j)}{1-\eta_j}r_j\right)\left[(I-\hat{\Omega})^{-1}\right]_{j0}} \tag{40}$$

where $\overline{\delta}_i \equiv \frac{\epsilon_i\left(1+\left(\frac{1-\delta_i}{\delta_i}\right)^{\frac{1}{\epsilon_i}}\left(\frac{G_i}{C_i}\right)^{\frac{1-\epsilon_i}{\epsilon_i}}\right)}{\frac{1}{\sigma}+\epsilon_i\left(\frac{1-\delta_i}{\delta_i}\right)^{\frac{1}{\epsilon_i}}\left(\frac{G_i}{C_i}\right)^{\frac{1-\epsilon_i}{\epsilon_i}}}$.

All in all, we have managed to obtain explicit expressions for all the terms featuring in the first order condition for optimal sector-specific government consumption, outlined in Proposition 1. Moreover, it can be easily verified that all the terms in the first order conditions are function of the following set of variables:

$$\left[\{x_j\}_{j=0}^{N}, \{G_j/C_j\}_{j=1}^{N}\right]. \tag{41}$$

In addition, recall the definition of spare capacity $S_j \equiv [1-f_j(x_j)]$, given in Definition 1, which implies a one-for-one mapping between S_j and x_j. We can therefore also write all the terms in the first order conditions are function of the following set of variables:

$$\varrho \equiv \left[\{S_j\}_{j=0}^{N}, \{G_j/C_j\}_{j=1}^{N}\right]. \tag{42}$$

In the next subsection we are going to find an first order approximation to the optimal sectoral government consumption condition around a specific value of ϱ.

4.3 Optimal fiscal policy: approximation near constrained efficiency

Making the dependence on ϱ explicit, consider the first order condition in Proposition 1 and take differentials from both sides:

$$d(\omega_i^G(\varrho))\left[1-MRS_i^{GC}(\varrho)\right]+\omega_i^G(\varrho)\left[0-d(MRS_i^{GC}(\varrho))\right] =$$

$$\sum_{t=0}^{N} d(\lambda_t(\varrho))\frac{d\log A_t(x_t)}{d\log x}(\varrho)\frac{d\log x_t}{d\log G_t}(\varrho)+\sum_{t=0}^{N}\lambda_t(\varrho)d\left(\frac{d\log A_t(x_t)}{d\log x}(\varrho)\right)\frac{d\log x_t}{d\log G_t}(\varrho)$$

$$+\sum_{t=0}^{N}\lambda_t(\varrho)\frac{d\log A_t(x_t)}{d\log x}(\varrho)d\left(\frac{d\log x_t}{d\log G_t}(\varrho)\right) \tag{43}$$

In this subsection we are interested in finding an approximation around the point of constrained efficiency, given by:

$$\varrho^* \equiv \left[\left\{ S_j^* \right\}_{j=0}^{N}, \left\{ (G_j/C_j)^* \right\}_{j=1}^{N} \right]. \tag{44}$$

where the constrained efficient sectoral spare capacity is $S_j^* \equiv 1 - f_i(x_i^*)$, and $(G_j/C_j)^*$ is defined by the condition $MRS_j^{GC}\left(\left(\frac{G_i}{C_i}\right)^*\right) = \left(\frac{\delta_i}{1-\delta_i}\right)^{\frac{1}{\epsilon_i}} \left(\left(\frac{G_i}{C_i}\right)^*\right)^{-\frac{1}{\epsilon_i}} = 1$, and corresponds to the sector-specific Samuelson level of government consumption.

Evaluating (43) at ϱ^*, it is apparent that the first term on the left hand side, as well as the the first and third terms on the right hand side are zero. This is because, on the one hand, $MRS_i^{GC}(\varrho^*) = 1$, and at the same time $\frac{d \log A_t(x_t)}{d \log x}(\varrho^*) = (1 - \eta_i) - \eta_i \gamma_i(x_i^*) = 0$, which follows directly from the definition of constrained efficient tightness in (37). Working out the remaining derivatives and evaluating at ϱ^* delivers the following approximate optimality condition for sector-specific government consumption near constrained efficiency:

Proposition 2 (Optimal policy near constrained efficiency)**.** *Around constrained efficiency, optimal deviations of sectoral government consumptions and sectoral spare capacities satisfy:*

$$\hat{gc}_i = \frac{\zeta_i}{1 - \delta_i} \times \underbrace{\left[\sum_{t=0}^{N} \lambda_t(\varrho^*) \frac{r_t}{1 - \eta_t} \hat{s}_t \right]}_{\text{Common component}} \tag{45}$$

where

$$\zeta_i \equiv \frac{\left(\frac{\delta_i}{1-\delta_i} \frac{1}{\epsilon_i} + \sigma \right)^{-1}}{\sum_{j=1}^{M} \omega_j^{CG}(\varrho^*) \left(\frac{\delta_j}{1-\delta_j} \frac{1}{\epsilon_j} + \sigma \right)^{-1}} \tag{46}$$

and $\quad \hat{gc}_i \equiv [\log(G_i/C_i) - \log(G_i/C_i)^*], \quad \hat{s}_t \equiv (S_t - S_t^*)/(1 - S_t^*)$.

The optimal deviation of $\log(G_i/C_i)$ from its Samuelson level is given by a product of an idiosyncratic and a common component. The idiosyncratic component is $\frac{\zeta_i}{1-\delta_i}$ and the crucial sector-specific factor which determines its magnitude is the elasticity of substitution across private and public provision of the sectoral good. In relative terms, sectors where private and public provision are closer substitutes should feature larger deviations of $\log(G_i/C_i)$ from its Samuelson level. The common component is a weighted sum of deviations of spare capacity where the weights are larger for sectors that: (i) have larger Domar weights; (ii) have smaller pass-through of marginal costs to prices; (iii) have lower matching function elasticity parameter.

Crucially, the approximate optimal policy condition near constrained efficiency does not feature any parameters related to the production process. Indeed, elasticities of substitution across inputs and input-output cost shares do not enter the approximate optimality condition in Proposition 2. This is the case for the following reason. As can be seen from (43), around constrained efficiency any movements in the Domar weights and the cross-elasticities of tightness are second order. And it is those terms that depend on the production function parameters.[6] In the next subsection we consider approximate optimal policy away from constrained efficiency, where those effects will be first-order.

4.4 Optimal fiscal policy: approximation away from constrained efficiency

[TO BE ADDED VERY SOON]

5 Conclusion

In this paper we develop a general theory of *optimal* sector-specific government spending, a policy practice we label *Keynesian Micromanagement*. To study optimal sector-specific spending, we build a novel multi-sector model which combines search-and-matching frictions at the level of individual goods with realistic heterogeneity in price rigidity, input-output linkages and elasticities of substitution in production. Our model generates involuntary spare capacity in all sectors of the economy, which is generally inefficient and should be corrected with sector-specific spending.

Our novel optimal policy principle directly exploits this equivalence between sectoral search frictions and endogeneous productivity changes. In particular, we show that sector-specific government spending should deviate from the frictionless benchmark given by the Samuelson rule to the extent and in the direction that sectoral spending affects aggregate measured total factor productivity. The latter can be calculated by using Hulten's theorem to aggregate sector-specific endogenous productivities.

[6] Notice that exactly at constrained efficiency, the cross-elasticity of tightness in (39) also does not depend on input-output cost shares. This is because $\hat{\omega}_{tj}(\varrho^*) = \omega_{tj}$, which means that $\left[(I-\hat{\Omega})^{-1}\right]_{t0} = \left[(I-\Omega)^{-1}\right]_{t0} = 1, \forall t$